2019 Advanced Algorithm Lec 1

2019年3月11日 17:02

Quick Sort.

$T(n)$.

Thm. $E(T(n)) = O(n \log n)$

$$E(T(n)) = \frac{1}{n} \sum_{k=1}^{n} \left(E(T(k-1)) + E(T(n-k)) + n-1 \right)$$

$$f(n) = \frac{1}{n} \sum_{k=1}^{n} \left(f(k-1) + f(n-k) + n-1 \right)$$

$$T(n) = T(X_n - 1) + T(n - X_n) + n - 1$$

X well-defined.

Event $(X_n = k)$

$\updownarrow$

Event $(T(k))$

$$E(T(n)) = \frac{1}{n} \sum_{k=1}^{n} \left(E(T(n)) \mid X_n = k \right).$$

$$= \frac{1}{n} \sum_{k=1}^{n} \left(E(T(k-1)) + E(T(n-k)) + n-1 \mid X_n = k \right).$$

$$= \frac{1}{n} \sum_{k=1}^{n} \left(E(T(k-1)) + E(T(n-k)) + n-1 \right).$$

Method 2.

$$X_{ij} = \begin{cases} 1 & P_{ij}. \\ 0. & 1 - P_{ij}. \end{cases}$$

$$T(n) = \sum_{1 \le i < j \le n} X_{ij}.$$

$E(X_{ij}) = P_{ij}.$

$i < j.$

$$E(T(n)) = \sum_{1 \le i < j \le n} P_{ij}.$$

$$P_{1n} = \frac{2}{n}$$

$i / j.$ $X_{ij} = 1$

$P_{1n} = \frac{2}{n}$

$P_{12} = 1$

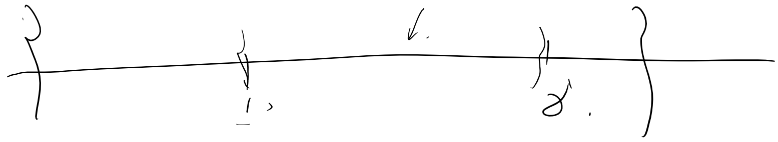
$P_{ij} = \frac{2}{j-i+1}$

i/j	$X_{ij}=1$
$i+1, i+2, \cdots, j-1$	$X_{ij}=0$
$1 \cdots i-1$	?
$j+1, \cdots, n$	?

$\{i, i+1, \cdots, j-1, j\}$

i/j $\quad X_{ij}=1 \quad \frac{2}{j-i+1}$

$i+1, \cdots, j-1 \quad X_{ij}=0 \quad \frac{j-i-1}{j-i+1}$



$1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$

$$\sum_{i<j\le n} \frac{1}{j-i+1}$$

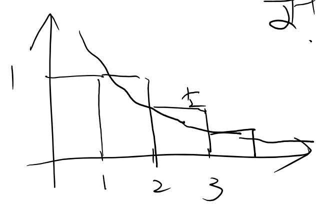
$$= 2\sum_{1\le i\le n-1} \sum_{j=2}^{n-i+1} \frac{1}{j} \le 2(n-1) \boxed{\sum_{j=2}^{n} \frac{1}{j}} \quad O(n\ln n)$$

$\approx \ln n$

~~$y=\ln x$~~

$y=\frac{1}{x}$ $\quad = \int_1^n \frac{1}{x} = \ln n - \ln 1 = \ln n$

$\frac{1}{2} + \cdots + \frac{1}{n} \le \int_1^n \frac{1}{x} = \ln n$



$1+\frac{1}{2}+\cdots+\frac{1}{n} \ge \int_1^n \frac{1}{x} = \ln n$

Min Cut

fix min cut C. $|C|=k$.

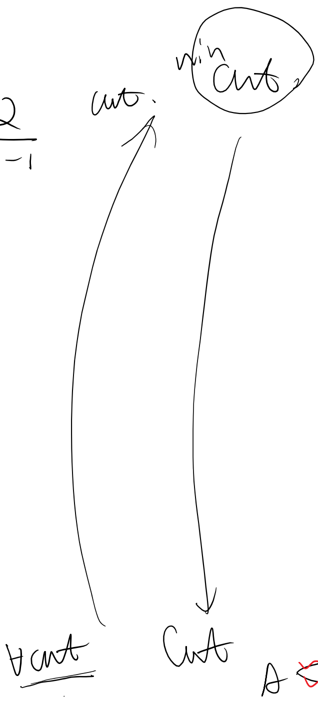
$\Pr(E_1 \wedge E_2 \cdots \wedge E_{n-1})$ $\quad E_i$: the i-th chosen edge is not in C

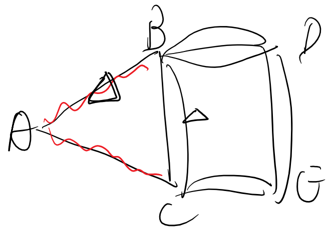
$$\Pr(E_1) = \frac{|E|-k}{|E|} \ge 1-\frac{2k}{nk} = 1-\frac{2}{n}$$

$$\underline{Pr(E_1)} = \frac{|E|-k}{|E|} \geq 1 - \frac{2k}{nk} = 1 - \frac{2}{n}.$$

$$\underline{|E|} \geq \frac{nk}{2}$$

$\times$ $Pr(E_2) \geq 1 - \frac{2}{n-1}$

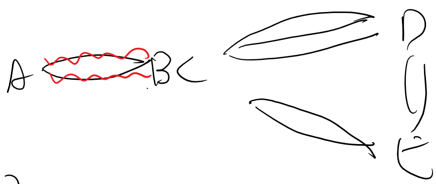




A | BCDE

AB | CDE

$\checkmark$ $Pr(E_2 \mid E_1) \geq 1 - \frac{2}{n-1}$



A | BC DE

$$Pr(E_i \mid \underline{E_1, \cdots, E_{i-1}}) \geq 1 - \frac{2}{n-i+1}$$

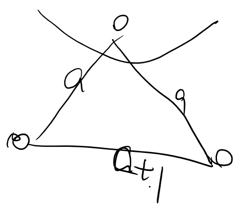
$$Pr(E_1 \wedge \cdots \wedge E_{n-2}) = Pr(E_1) \cdot Pr(E_2 \mid E_1)\, Pr(E_3 \mid E_1 E_2) \cdots$$

$$\geq \left(1 - \frac{2}{n}\right)\left(1 - \frac{2}{n-1}\right) \cdots \left(1 - \frac{2}{3}\right)$$

$$= \frac{\cancel{n-2}}{n} \cdot \frac{\cancel{n-3}}{n-1} \cdot \frac{\cancel{n-4}}{\cancel{n-2}} \cdots \frac{2}{\cancel{4}} \cdot \frac{1}{\cancel{3}}$$

$$= \Theta\left(\frac{1}{n^2}\right).$$

$$Pr(\quad) = \Omega\left(\frac{1}{n^2}\right)$$



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repeat n^2 times.

successful probability: $1 - \left(1 - \frac{1}{n^2}\right)^{n^2} \sim 1 - \frac{1}{e}$.

sum.

Successful.

						sum
1	2	3	7	8	0	21
	6	0	0	0	0	6
		0	9	0	0	9
			0	10	0	10
				0	11	11
					0	0

$|E| = 1+2+3+7+8+6+9+10+11 = 57$